Student Instruction Sheet: Unit 1, Lesson 3

Common Factoring

Suggested Time: 35 minutes

What's important in this lesson:

In this lesson, you will learn how to find the greatest common factor, using algebra and/or a graphing calculator.

Complete the following steps:

1. Read through the lesson portion of the package on your own.
2. Complete the exercises.
3. Check your answers with the Answer Key that your teacher has.
4. Seek assistance from the teacher as needed.

Hand in the following:

1. Student Handout

Questions for the teacher:

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Vocabulary

A **factor** of a number is any number that will divide evenly into that number.
For example, the factors of 12 are 1, 2, 3, 4, 6, and 12.

A **factor** of a monomial is any monomial that will **divide evenly** into that monomial.
For example, the complete list of possible factors of $6m^3$ is

$1, 2, 3, 6, m, 2m, 3m, 6m, m^2, 2m^2, 3m^2, 6m^2, m^3, 2m^3, 3m^3, 6m^3$

The **<u>G</u>reatest <u>C</u>ommon <u>F</u>actor** of a group of terms (or polynomial) is the largest monomial that will divide evenly into every term in the group of terms (or polynomial). We often use the short form **GCF.**

The definitions above all use the word “factor” as a noun (a thing). However, the word “factor” can also be used as a verb (an action).

If you are asked to **factor** an expression, you are being asked to rewrite that expression as the product of two or more smaller factors that can be multiplied together to get back to the original expression.

If you are asked to **common factor** a polynomial, you are being asked to find the greatest common factor of all the terms and then rewrite the polynomial as a product that starts with the GCF.

<u>Topic 1: Find the GCF of a Group of Numbers</u>

To find the GCF of a group of numbers, you must find the single largest number that will divide evenly into every number. For some numbers, you can do this in your head. For example, the GCF of 5 and 10 is 5. The GCF of 4 and 6 is 2. But what about the GCF of 36 and 84?

Fortunately, we can use a graphing calculator to help us out.

On your graphing calculator, hit the MATH key and then the right arrow.

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Arrow down until you highlight **9:gcd.**

Hit ENTER. You will see **gcd.**

Input the numbers you are working with, and put a comma between each number.

Hit ENTER to get your answer.

The greatest common factor will appear on the right side of the screen.

If you need to find the GCF of three numbers, you will have to find the GCF of the first two and then repeat the routine to find the GCF of the third number and the GCF of the first two numbers.

If you don't have a graphing calculator available, you should know how to find the GCF by hand. To do this, you must write each number as a product of its **prime** factors. (This is faster than writing all of the factors.) Once each number is factored, circle all of the matching factors and multiply them together to get the GCF.

For example:

36 = 2 x 18 = 2 x 2 x 9 = 2 x 2 x 3 x 3
84 = 2 x 42 = 2 x 2 x 21 = 2 x 2 x 3 x 7

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In this example, the matching factors are two 2's and a 3. When they are multiplied together, they produce the GCF of 12 for the two numbers 36 and 84.

A prime number can be divided only by 1 and itself. The prime numbers up to 50 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47. They have no other factors.

Keep trying to divide by each prime number until it doesn't work any more, and then try the next prime number. Once you get a prime number at the end of your list, you will know that you have finished.

Try the following examples.

1. Write each number as the product of its prime factors.

[a] 24 = ____________________________

[b] 42 = ____________________________

[c] 120 = ____________________________

2. [a] What is the greatest common factor of 24, 42, and 120?

[b] Explain how you were able to use the information in Question 1 to answer Question 2 [a].

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Topic 2: Finding the GCF (Greatest Common Factor) of a Group of Terms

You have just learned how to find the GCF of the numbers for each term. That's the hardest part. To deal with the variables in their terms, just apply the following two rules:

1. If a variable (letter) is found in **all** of the terms, it should be included in the GCF.
2. For each variable included in the GCF, use the **smallest** exponent found for that variable in the original terms.

Examples:

What is the GCF of $3x^2$, 6x, and 9?

Looking just at the numbers 3, 6, and 9, the highest match is 3, so the GCF is 3. The variable *x* is found only in two of the terms, so it is **not** included in the GCF.

What is the GCF of $4x^3$, $10x^2$, and 20x?

Looking just at the numbers 4, 10, and 20, the highest match is 2; the GCF = 2.
The variable ***x*** is found in **all three terms**, so it must be included in the GCF.
The smallest exponent is the invisible "1" on the *x* in 20*x*.
The GCF of the three terms is 2*x*, which is the largest monomial that will divide evenly into all of the terms.

Try the following.
Identify the greatest common factor (GCF) for each group of terms.

[a] 15, 40

$\Rightarrow$ GCF is ____________

[b] $8x^2$, $36x$

$\Rightarrow$ GCF is ____________

[c] $6x^2$, $18x$, 24

$\Rightarrow$ GCF is ____________

[d] $4x^2$, 14x, 8x

$\Rightarrow$ GCF is ____________

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Topic 3: Rewriting a Polynomial as a Product That Contains the GCF of All Its Terms.

When a question asks you to **factor** or **common factor** a polynomial:

Step 1. Find the GCF of all the terms and write it below the original polynomial.
Step 2. Open a bracket after the GCF.
Step 3. Divide each of the original terms by the GCF, and write each result inside the bracket.

Examples:

Rewrite each expression as a product of the greatest common factor and a polynomial.

$8x + 12$
$= 4(2x + 3)$

$6x^2 - 15x$
$= 3x(2x - 5)$

$x^3 + 8x^2 - 7x$
$= x(x^2 + 8x - 7)$

Steps:
4 is GCF, $8x \div 4 = 2x$, $12 \div 4 = 3$

Steps:
x is GCF, $x^3 \div x = x^2$, $8x^2 \div x =$ *8x, etc.*

Here are some final questions for you to try.

Rewrite each expression as a product of the Greatest Common Factor and a polynomial.

[a] $10x^2 - 15x$
=

[b] $5x^2 + 25x + 30$
=

[c] $4x^2 - 16$
=

[d] $6x^4 + 21x^2$
=

[e] $x^3 - 5x^2 + 10x$
=

[f] $5x^3 - 45x$
=