Student Instruction Sheet: Unit 2, Lesson 3

Linear Systems

Suggested Time: 75 minutes

What's important in this lesson:

In this lesson, you will learn how to solve for the point of intersection between two lines.

Complete the following steps:

1. Read through the lesson portion of the package on your own.
2. Complete the exercises.
3. Check your answers with the Answer Key that your teacher has.
4. Seek help from the teacher if you need it.
5. Complete the Assessment and Evaluation and hand it in.

Hand in the following:

1. Student Handout
2. Assessment and Evaluation

Questions for the teacher:

Student Handout: Unit 2, Lesson 3

Vocabulary

Each equation in the system is actually a linear relation and will define a straight line. When we **solve** a system of equations, we are finding the coordinates of the **intersection point of the two lines**. This is the **point that is "common"** to both lines.

Algebraic methods use only equations to find the *x* and y values for the common point (although a graph could be drawn from such an equation).

If we look at a table of values, the common point occurs where the same x and y values are found in each table.

y = x + 6

x	y
0	6
1	7
2	8
3	9
4	10

y = 2x + 12

x	y
0	12
1	10
2	8
3	6
4	4

Example1- point (2, 8) is on both lines.

y = 3x – 3

x	y
1	0
2	3
3	6
4	9
5	12

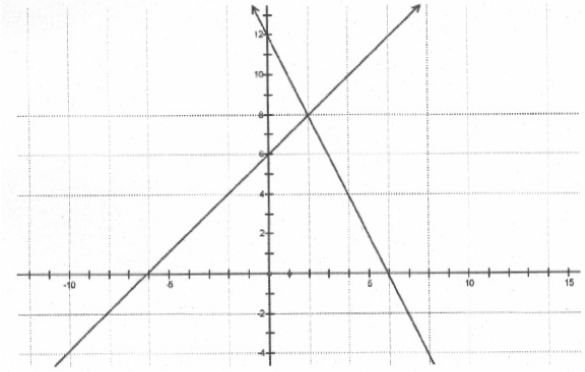
y = 2x

x	y
1	2
2	4
3	6
4	8
5	10

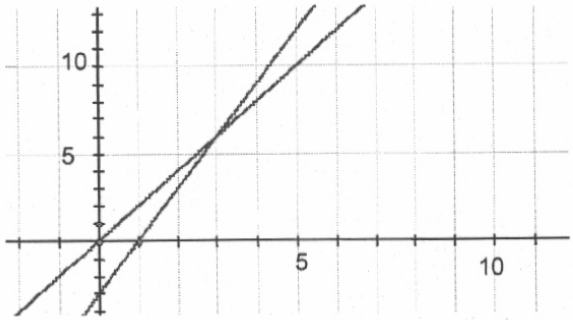
Example 2- point (3, 6) is on both lines.

Graphical methods use graphs to find the intersection point.

When we graph two lines the solution is the point where the lines cross.



In this example (,) is the intersection of the system y = x + 6 and y = – 2x + 12



In this example (,) is the intersection. of the system y = 3x – 3 and y = 2x

If we graph the lines accurately, it doesn't matter whether we see the common point in the tables because it will show up on the graph.

Student Handout: Unit 2, Lesson 3

Topic 1: Graphical Methods Example

Cellphone company A charges $35 per month plus 10 cents per minute. Cellphone company B charges $45 per month and 8 cents per minute. For how many minutes of calling time will the monthly cost be the same for both companies?

Step1: Write an equation for the total cost, "C," of using company A for "*x*" minutes.

$$C = 0.10x + 35$$

Step2: Write an equation for the total cost, "C," of using company B for "*x*" minutes.

$$C = 0.08x + 45$$

Step 3: Graph the two equations for the total cost of each phone for up to 600 minutes.

Title:

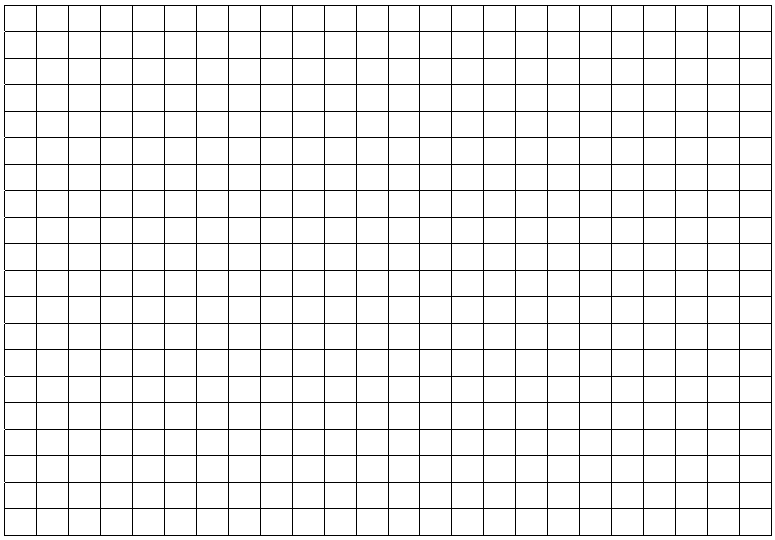
COST IN DOLLARS

80

60

40

20



100 200 300 4 00 500

TIME IN MINUTES

Sketch the result.

Window settings
Xmin 0
Xmax 600

Xscl 50
Ymin 0
Ymax 100

Yscl 10
Xres 1

To find the intersection, use the Calculate menu.

- Press 2ND TRACE for the [CALC] menu.
- Select **5: intersect**
- You will be presented with the graph and a series of questions in the lower left-hand corner.
- "**First curve?**" The cursor will be flashing and positioned on the first equation. Press ENTER.
- "**Second curve?**" The cursor will be flashing and positioned on the second equation. Press ENTER
- "**Guess?**" Press ENTER
- "**INTERSECTION**." The *x*- and y-coordinates of the point of intersection will appear at the bottom of the screen.

So we see that the point of intersection is (,).

Student Handout: Unit 2, Lesson 3

Topic 2: Algebraic Method 1 – Substitution (This method is also called *comparison* if both equations are in the "y =" form.)

Example 1

Find the solution of:

$y = 3x + 4$ ①

$2x + y = 9$ ②

Step 1: Sub ① into ② (This means replace the "y" in equation 2 with the right-hand side of the equation for y in equation 1.)

$2x + (3x + 4) = 9$

Step 2: Solve for x.

$5x + 4 = 9$

$5x = 9 - 4$

$5x = 5$

$x = 1$

Step 3: Substitute the value of *x* from Step 2 back into equation ① to get the y value.

$y = 3(1) + 4$

$y = 3 + 4$

$y = 7$

Step 4: Make sure that the point is on the other line as well.
(L.S. indicates the left side of the original equation.
R.S. is the right side of the original equation.)

Check (1, 7) in ②

L.S.	R.S
$2x + y$	9
$= 2(1) + 7$	
$= 2 + 7$	
$= 9$	

L.S. = R.S.

∴ the solution is (1,4)

Topic 2: Algebraic Method 1 – Substitution, continued

Example 2

Solve this system of equations to find the point of intersection.

$$y = 2x - 3 \quad ①$$
$$x - 3y = -1 \quad ②$$

Step 1:
Sub ① into ② to replace y

$$x - 3(2x - 3) = -1$$
$$x - 6x + 9 = -1$$

Step 2:
Solve for x

$$-5x = -1 - 9$$
$$-5x = -10$$
$$x = 2$$

Step 3:
Sub $x = 2$ into ①

$$y = 2(2) - 3$$
$$y = 4 - 3$$
$$y = 1$$

Step 4:
Check (2, 1) in ②

L.S.	R.S.
x – 3y	-1
= (2) – 3(1)	
= 2 – 3	
= -1	

L.S. = R.S.

∴ the solution is (2, 1)

Student Handout: Unit 2, Lesson 3

Topic 3: Algebraic Method 2 – Elimination (In this method, we add or subtract the equations to eliminate (remove) one of the variables)

Example 1 Solve $2x + 5y = 17$ ①

$4x - 5y = -11$ ②

Step 1 – Check to see whether *one of the variables* can be eliminated by simply adding or subtracting the equations. In this case, adding will eliminate the variable y.

Step 2 – Add or subtract the equations.

$$\begin{array}{rl} & 2x + 5y = 17 \\ & \underline{4x - 5y = -11} \\ \text{Add} & 6x \quad = \quad 6 \end{array}$$

Step 3 – Solve for the remaining variable. $x = 1$

Step 4 – Substitute the value found for x in equation ① to find the other variable.

$$\begin{aligned} 2(1) + 5y &= 17 \\ 2 + 5y &= 17 \\ 5y &= 17 - 2 \\ 5y &= 15 \\ y &= 3 \end{aligned}$$

Step 5 – Check that the point is on the other line as well.

Check (1,3) in equation ②

L.S.	R.S.
2x + 5y	17
= 2(1) + 5(3)	
= 2 + 15	
= 17	

L.S. = R.S.

∴ Solution is (1,3)

Topic 3: Algebraic Method 2 – Elimination, continued

Example 2

Solve $x + y = -1$ ①

$3x – 4y = 25$ ②

Step 1 – In this case, neither adding nor subtracting will eliminate a variable, since none of the coefficients (numbers in front of the variable) are the same. We can multiply every term of one equation by a constant to make the variables the same.

Multiply equation one by 3 to make the *x* coefficients the same in both equations.

$3x + 3y = -3$ ①

Step 2 $3x + 3y = -3$ new ①

Subtract $3x – 4y = 25$

$7y = -28$

Step 3 $y = -4$

Step 4 Sub y = -4 into original equation ①

$x + (-4) = -1$
$x – 4 = -1$
$x = -1 + 4$
$x = 3$

Step 5 Check (3, -4) in equation ②

L.S.	R.S.
3x – 4y	25
= 3(3) – 4(-4)	
= 9 + 16	
= 25	

L.S. = R.S.

∴ Solution is (3, -4)

Assessment and Evaluation: Unit 2, Lesson 3

1. Use the graphing calculator to find the point of intersection of each of the following. Draw a sketch, and state your window settings for each question.

[a] $y = 3x + 2$
$y = -2x + 52$

[b] $y = 6x + 5$
$y = 3x – 5$

2. Using the method of substitution, solve the following system of equations.

[a] $y = 3x – 2$
$y = - 2x – 7$

[b] $y = 2x + 4$
$2x + y = 12$

Assessment and Evaluation: Unit 2, Lesson 3

3. Use the method of elimination to solve the following system of equations.

[a] $2x + y = 5$

$3x - y = 10$

[b] $3x - 2y = 5$

$2x - 4y = 6$

Student Instruction Sheet: Unit 2, Lesson 3

Reflective Activity, Unit Two

Suggested Time: 35 minutes

What's important in this lesson:

It is important that you work carefully through the questions on this page. These questions have been designed to review the material in this unit.

Complete the following steps:

1. Answer all questions provided.
2. If you have any questions, ask the teacher.
3. Check your answers with the teacher.

Hand in the following:

Reflective Activity:
Look carefully through this unit. There were 3 Lessons. Included in each lesson were numbered Topics. Write in the headings for each, as in this example:
Lesson 1: _____

Topic 1: _____ and so on.

Write a brief description of what you learned in each topic.

Questions for the teacher: