Student Instruction Sheet: Unit 3, Lesson 1

Parabolas
$y = a(x - p)^2 + q$

Suggested Time: 75 minutes

What's important in this lesson:

In this lesson, you will learn what each part of the parabola is all about. You will learn how to find the vertex and the axis of symmetry, and how to determine whether there is a stretching or compressing factor.

Complete the following steps:

1. Read through the lesson portion of the package on your own.
2. Complete the exercises.
3. Check your answers with the Answer Key that your teacher has.
4. Seek assistance from the teacher as needed.
5. Complete the Assessment and Evaluation and hand it in.

Hand in the following:

1. Student Handout
2. Assessment and Evaluation sheet

Questions for the teacher:

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Parabolas Background

We have learned that any relation of the form $y = ax^2 + q$ is called a quadratic relation and that it graphs as a parabola. This means that whenever we see a relation that has the exponent 2, we can expect to get a parabola in our graph. In this lesson, we are going to use a graphing calculator to investigate the importance of:

- the number written in front of the x^2; we'll call it "a"
- the number that is added to or subtracted from the x before we square it ; we'll call it p; this value is always inside a bracket
- the number that is added **after** we do the squaring; we'll call it q

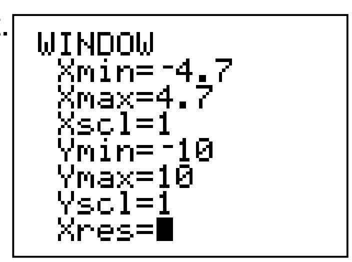
Topic 1: The Meaning of "a"

The purpose of this investigation is to predict the shape and direction of a parabola defined by $y = ax^2$ based on the value of "a."

Use the TABLE function of the calculator to complete the following table of values. (Press "y =" then input the function, press "2nd", then press GRAPH. Use up and down arrows as needed to see the requested input values.)

x	$y = x^2$	$y = 2x^2$	$y = 3x^2$	$y = 4x^2$
- 4				
- 3				
- 2				
- 1				
0				
1				
2				
3				
4				

Press the window key and set it to match the screen at the right.
Note: To input a negative sign, you must use the grey key below the "3," which has a negative sign in brackets. If you try to use the blue subtraction key on the right side, you will get an error message.



Now copy each graph onto the following page.

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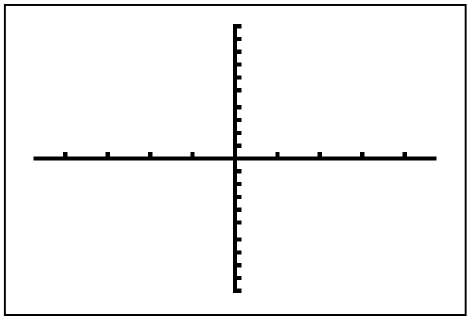
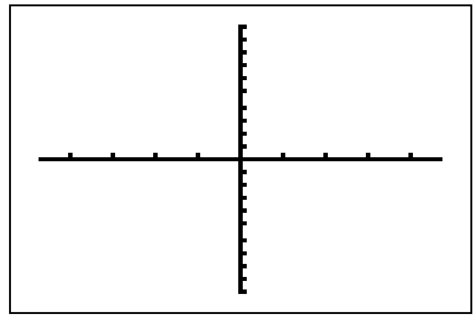
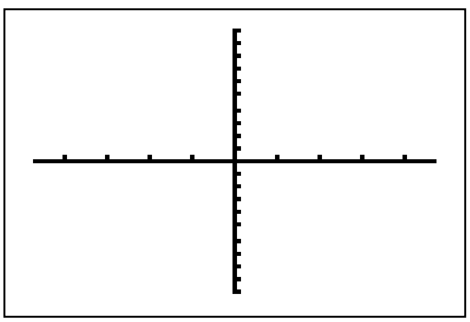
$y = x^2$



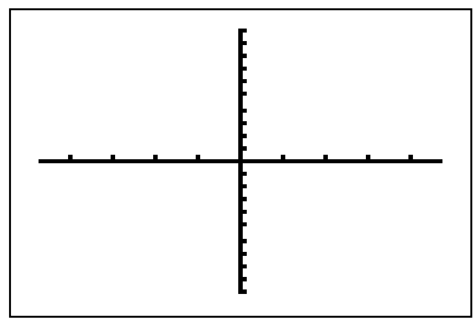
$y = 2x^2$



$y = 3x^2$



$y = 4x^2$

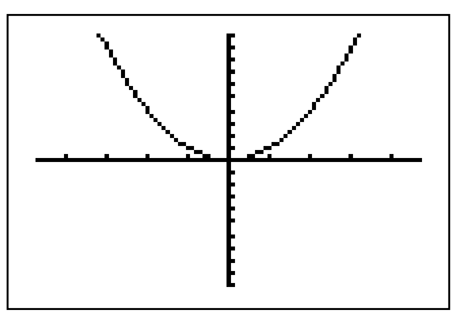


1. [a] What were the coordinates of the vertex for all four parabolas?

   [b] What effect did the "a" value have on the location of the vertex?

2. As the values chosen for "a" got larger, what happened to the shape of the parabola?

The graph below belongs to $y = x^2$. On the same graph, sketch your **prediction** for the graph of $y = \frac{1}{2}x^2$.(Use the calculator to check your prediction!)



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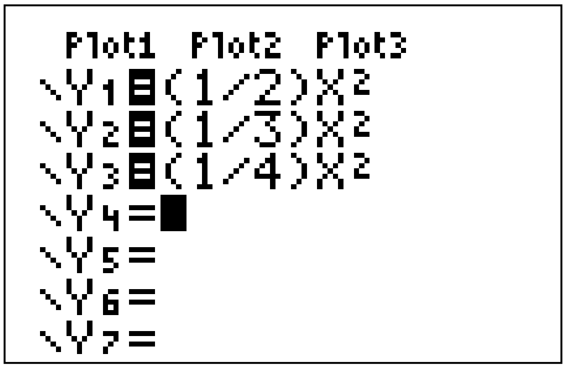
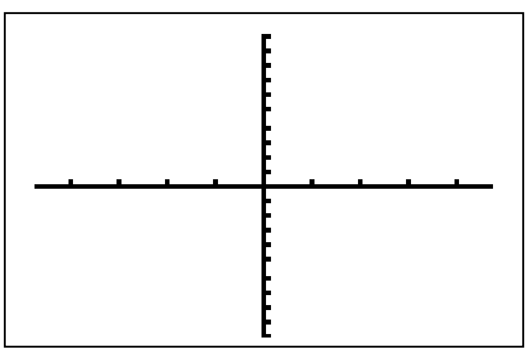
Input the relations $y = \frac{1}{2}x^2$, $y = \frac{1}{3}x^2$ and $y = \frac{1}{4}x^2$ into your calculator. Start with the "y =" key. To input the fractions, key a left bracket, a 1, the division button, the denominator, and a right bracket. The input screen should look like the one below.



Use the TABLE function of the calculator to complete the following table of values. Round all numbers to one decimal place, if necessary.

x	$y = x^2$	$y = \frac{1}{2}x^2$	$y = \frac{1}{3}x^2$	$y = \frac{1}{4}x^2$
- 4	16			
- 3	9			
- 2	4			
- 1	1			
0	0			
1	1			
2	4			
3	9			
4	16			

On the single set of axes below, graph the three parabolas for the relations that you completed above. Label each curve with its equation.



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3. [a] If the value of "a" is positive, the parabola must open __________.

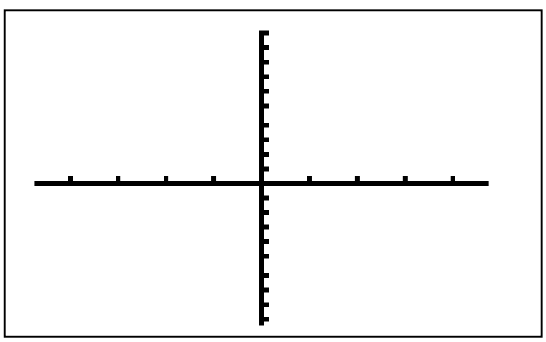
   [b] If the value of "a" is larger than 1, the parabola will be __________ the graph of $y = x^2$.

   [c] If the value of "a" is bigger than 0, but smaller than 1, the parabola will still open __________, but the parabola will be _________than the graph of $y = x^2$.

Input the relations $y = -x^2$, $y = -2x^2$ and $y = \frac{-1}{2}x^2$, into the graphing calculator.

Fill in the table of values and sketch the parabolas.

x	$y = -x^2$	$y = -2x^2$	$y = -\frac{1}{2}x^2$
-3			
-2			
-1			
0			
1			
2			
3			



4.[a] If the value of "a" is negative, the parabola for $y = ax^2$ must open __________.

[b] Describe how the graphs of $y = 3x^2$ and $y = -3x^2$ will compare to the basic parabola for $y = x^2$.

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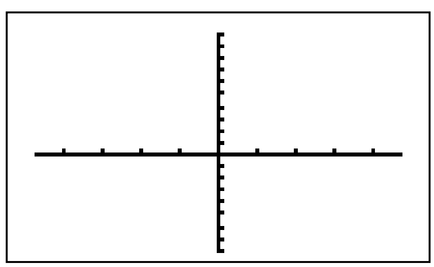
Topic 2: The Meaning of "q"

From the first investigation, we learned that the value of "a" controls the size and shape of the parabola. We should also note that the vertex of every quadratic relation of the form, $y = ax^2$ must be located at the origin (0, 0).

The big question for this investigation is "What happens if we add or subtract a number after we do the squaring and multiplying?" We could ask the same question mathematically by saying, "How is the graph of $y = ax^2 + q$ different from the graph of $y = ax^2$?"

Use the TABLE function on the graphing calculator to complete the table of values. Then graph each set of relations on the axes provided.

x	$y = x^2$	$y = x^2 + 2$	$y = x^2 - 4$
-3			
-2			
-1			
0			
1			
2			
3			



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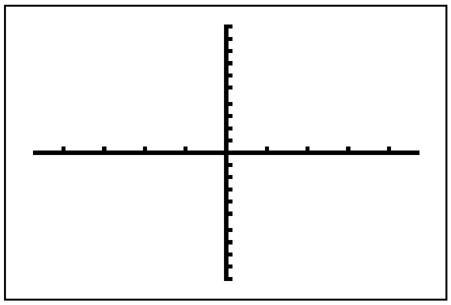
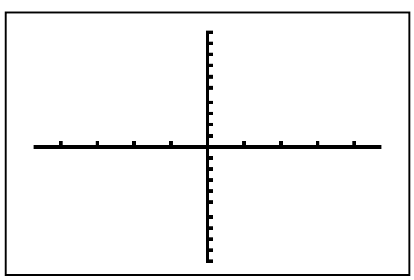
x	$y = -x^2$	$y = -x^2 + 5$	$y = -x^2 - 3$
-3			
-2			
-1			
0			
1			
2			
3			



x	$y = 2x^2$	$y = 2x^2 - 3$	$y = 2x^2 - 6$
-3			
-2			
-1			
0			
1			
2			
3			



1. What happens to the graph of $y = ax^2$ when we add a positive value for “q”?

2. What happens to the graph of $y = ax^2$ when we add a negative value for “q”?

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3. What are the coordinates of the vertex for the graph of $y = ax^2 + q$?

4. State the coordinates of the vertex for each of the following relations.

[a] $y = x^2 + 3$

(,)

[b] $y = 2x^2$

(,)

[c] $y = -x^2 + 4$

(,)

[d] $y = \frac{1}{2} x^2 - 5$

(,)

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Topic 3: The Meaning of "p"

In the first investigation, we saw how changing the value in front of the x^2 changed the shape and direction of the parabola.

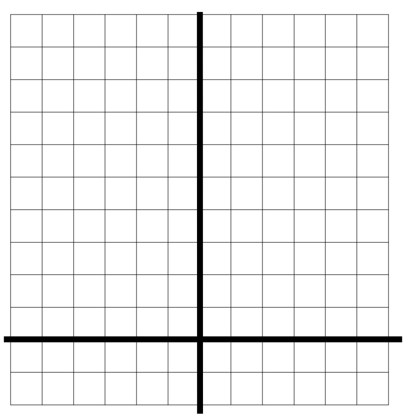
In the second investigation, we saw how adding or subtracting a number at the end, shifted the parabola up or down, without changing its shape.

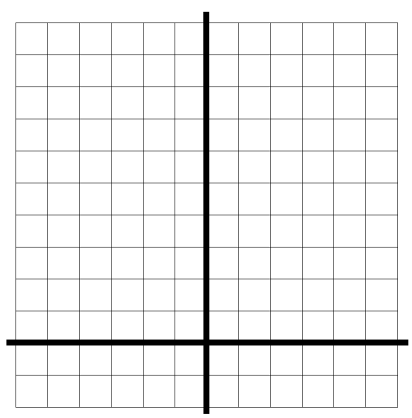
In this investigation, we will see how we can move a parabola to the right or left. Complete each of the following tables of values **by hand**, and graph the results neatly on the grids provided. The previous graphs were rough sketches; these graphs should be accurate.

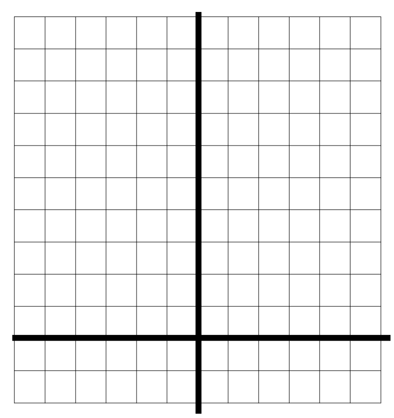
x	$y = x^2$
-3	
-2	
-1	
0	
1	
2	
3	

x	$y = (x - 2)^2$
-1	
0	
1	
2	
3	
4	
5	

x	$y = (x + 3)^2$
-6	
-5	
-4	
-3	
-2	
-1	
0	







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1. What do you notice about the output column for all of the tables of values?

2. Which way, and how far, did the basic parabola shift when we **subtracted** a value inside the bracket?

3. Which way, and how far, did the basic parabola shift when we **added** a value inside the bracket?

4. State the coordinates of the vertex for each relation. (Refer to your graphs.)

   [a] $y = (x - 2)^2$

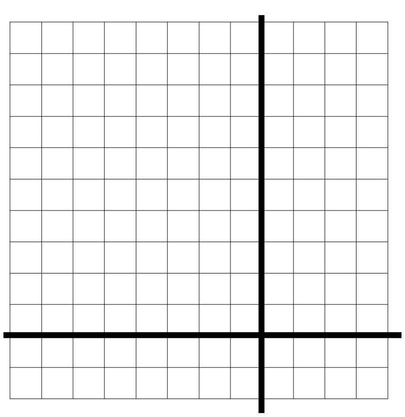
   (,)

   [b] $y = (x + 3)^2$

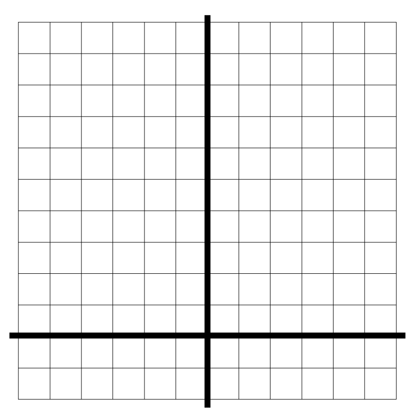
   (,)

5. Without preparing a table of values, sketch the graph of each function. Use a graphing calculator to check your work.

   [a] $y = (x + 4)^2$



   [b] $y = (x - 1)^2$



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6. Suppose that you had to explain to another student how to set up tables of values to obtain the graphs in Question 5. Write a set of instructions explaining how to use the given equation to find the vertex, and how to choose the input values, based on where the vertex is located.

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Topic 4: Interpreting Relations in the Form $y = a(x - p)^2 + q$ in Problems

If we have a quadratic relation written in the above form, we know that the vertex of the parabola must be located at (p, q). The vertex is either the *highest or lowest* point of the parabola, depending on which way the parabola opens.

If we have a quadratic relation that is being used to represent a real situation, the *q value of the vertex will give the maximum or minimum value* of the relation and the *p value will reflect which input value will give this* maximum or minimum.

For example, the relation $h = -5(t - 0.5)^2 + 11.25$ can be used to model the height of a diver who dives from a 10-metre platform.
The variable, h, represents the height of the diver above the water.
The variable, t, represents the time elapsed after the diver jumps.

By writing the relation this way, we can see that the vertex is (0.5, 11.25), (p,q). This means that the highest point above the water is 11.25 metres and that this will occur 0.5 seconds after the diver jumps. We know that this value will be a *maximum* height because the negative value for “a” means that the parabola opens down.

Have a quick look at Topic 4 from Unit 1, Lesson 2. If we expand and simplify this function, we see that it could also be written as $h = -5t^2 + 5t + 10$. When the function is written in this form, we can quickly get the height (when t = 0), which will be 10 m.

Now we can see that the function describes a diver who jumps from a 10 m platform, reaches a maximum height of 1.25 m above the platform (or 11.25 m above the water), and then dives downward into the pool.

Assessment and Evaluation: Unit 3, Lesson 1

1. Complete the following table.

Quadratic Function	Coordinates of Vertex	Direction of Opening of Parabola
$y = x^2$		
$y = -3x^2$		
$y = 2x^2 + 5$		
$y = (x + 4)^2$		
$y = (x - 3)^2 + 1$		
$y = -5(x + 2)^2$		

2. Write the function that matches each description. Check your functions with a graphing calculator to ensure that your functions work.

   [a] The function has the same shape as $y = 2x^2$. It opens down and has its vertex at (5, -4).

   [b] The function has the same shape as $y = \frac{1}{2}x^2$. It opens up and has its vertex at (0, 3).

   [c] The function has the same shape as $y = -3x^2$. It opens up and has its vertex at (4, 0).

3. Describe (in words) what you would have to do to the graph of $y = x^2$ to make it match the graph of each given function.

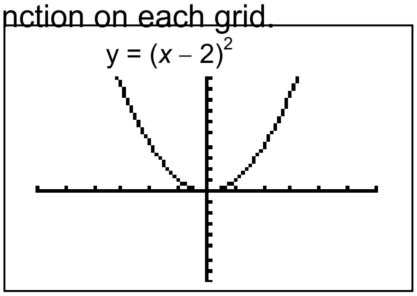
   [a] $y = 2x^2$

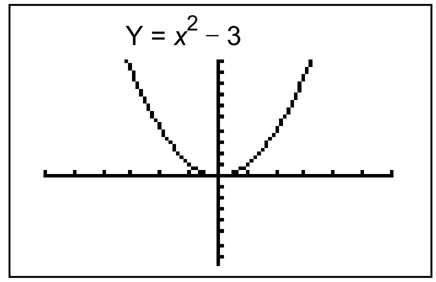
   [b] $y = (x + 1)^2$

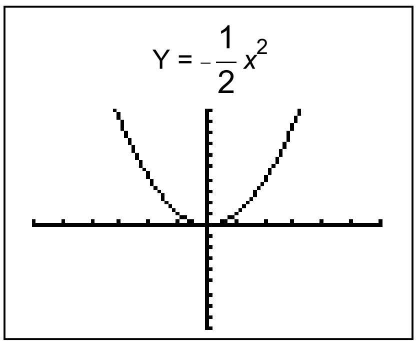
   [c] $y = x^2 + 5$

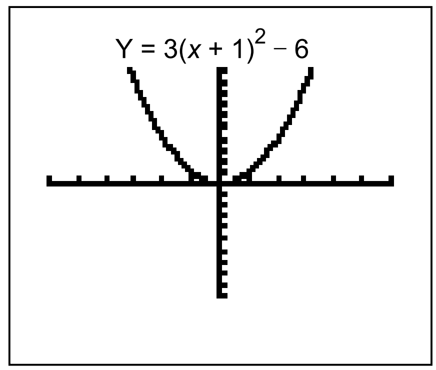
Assessment and Evaluation: Unit 3, Lesson 1

4. Each of the following grids has a graph of $y = x^2$. Draw a sketch of the given function on each grid.

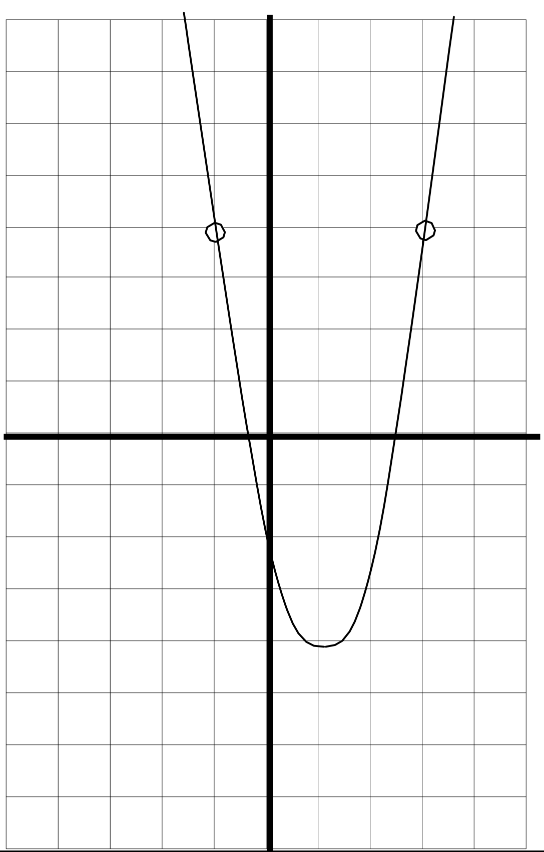








5. The following graph is a parabola.



[a] State the coordinates of the vertex.

[b] State the value of "a."

[c] Write the equation for the function in the form $y = a(x - p)^2 + q$.