Student Instruction Sheet: Unit 3, Lesson 3

Solving Quadratic Relations

Suggested Time: 75 minutes

What's important in this lesson:

In this lesson, you will learn how to solve a variety of quadratic relations.

Complete the following steps:

1. Read through the lesson portion of the package on your own.
2. Complete the exercises.
3. Check your answers with the Answer Key that your teacher has.
4. Seek assistance from the teacher as needed.
5. Complete the Assessment and Evaluation and hand it in.

Hand in the following:

1. Student Handout
2. Assessment and Evaluation sheet

Questions for the teacher:

Student Handout: Unit 3, Lesson 3

Background
This lesson covers the application of some skills already covered, but also includes a few new skills related to solving **quadratic equations**.

Solving Quadratic Equations
To solve a quadratic equation, we have to find all values of *x* that can be substituted into a given quadratic relation to make it equal a given number.

We will look at a few simple types of quadratic equations that can be solved by hand. We will then look at a more general approach to solving quadratic equations, which can be done using a graphing calculator.

Topic 1: Solving Quadratic Equation of type: "$x^2 = k$"

In this case, we must find what values squared make a given number.

This is the easiest possible type of quadratic equation to solve. The first thing to keep in mind is that it could have **2, 1, or 0 possible solutions.**

If k = 0, the equation is $x^2 = 0$, which has only **one possible solution**, x = 0.

If $k < 0$, the equation could be $x^2 = -5$, which has **no possible solution** because the value of x^2 can never be smaller than 0. In this case, we say that the equation has no solution.

If k > 0, the equation could be $x^2 = 25$, which would have **two possible solutions**.
The first would be *x* = 5, which we find by taking the square root of 25. The second would be *x* = -5, because if we square a negative number, we get a positive answer.

In general, if we have $x^2 = k$, then the solutions are $x=\sqrt{k}$ and $x = \sqrt{-k}$, as long as the value of k isn't negative. If k doesn't have an exact square root, we need to use a calculator and then round off the answer.

For example, if we have $x^2 = 20$, the solutions are $x= \sqrt{20}$ and $x = \sqrt{-20}$

When rounded, x = 4.47 and x = -4.47.

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Topic 2: Solving Quadratic Equation of type: "$ax^2 = k$"

This is similar to the first type of quadratic equation, except that there will be a number in front of x^2.

Example 1

Solve $4x^2 = 25$ — To start, we divide both sides of the equation by the number in front of the x^2.

$x^2 = 25 \div 4$

$x^2 = 6.25$ — Now it's just like first type, so we take the square root of 6.25.

$x = 2.5$ or $x = -2.5$

Example 2

Solve $x^2 - 10 = 0$ — Put the number on the right side of the equation.

$x^2 = 10$

$x = 3.16$ or $x = -3.16$ — Now use a calculator and finish it as first type.

Example 3

Solve $x^2 - 36 = 0$ — We know that the square root of 36 is 6. This means that we can **factor** the left side, which is a difference of squares.

$(x + 6)(x - 6) = 0$

From here, we must multiply the two brackets together to get a value of 0. This means that we need one bracket to take on a value of 0.

For the first bracket to be 0, we solve

$x + 6 = 0$

$x = -6$

For the second bracket to be 0, we solve

$x - 6 = 0$

$x = 6$

$x = 6$ or $x = -6$

Note: The second equation could have been done without factoring by rewriting it as $x^2 = 36$!

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Topic 3: Solving Quadratic Equations with a Graphing Calculator

The following are some examples of quadratic equations that require a graphing calculator to solve.

$x^2 + 5x + 5 = 12$

$2(x - 3)^2 - 7 = 0$

$x^2 + 5x = 3x$

To solve any quadratic equation using a graphing calculator, follow these steps:

1. Press ZOOM, then “6”. This resets the window to a fairly normal scale.
2. Press “Y=” and input the left side of the equation as a function for Y_1.
3. Input the right side of the equation as a function for Y_2.
4. Press GRAPH. If you see two graphs that are meeting, go to step 5.
5. If you can only see one graph, or if you can’t see the points where the graphs cross, press ZOOM, “3,” ENTER. This should fix things. If it doesn’t, try again. If it still doesn’t work, talk to your teacher.
6. Press “2nd”, then TRACE, then “5.”
7. Press the left arrow until the cursor is to the left of the **left-hand intersection point**. Then press ENTER **three times**.
8. Read the *x* value from the bottom of the screen. This is your first possible solution. The second value is just the y-coordinate of the point where the graphs cross. You weren’t asked for it, so don’t include it!
9. Press “2nd”, then TRACE then “5.”
10. Press the right arrow until the cursor is just left of the **right-hand intersection point**. Then press ENTER **three** times.
11. Read the *x* value from the bottom of the screen. This is your second possible solution.

Using a graphing calculator, follow these steps to see if you can match the screens on the next page.

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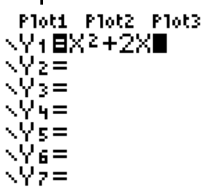
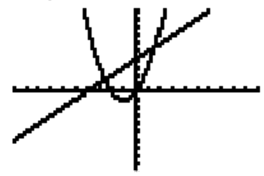
Example 1: Solve $x^2 + 2x = x + 4$

Step 1



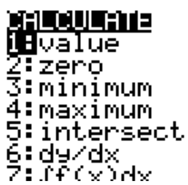
Step 2

Step 3

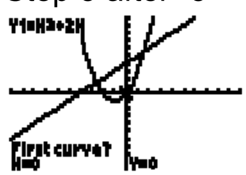


No Step 4 needed

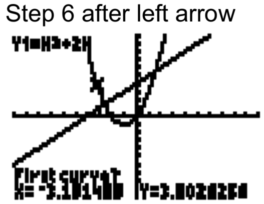
Step 5 after "2nd" and TRACE



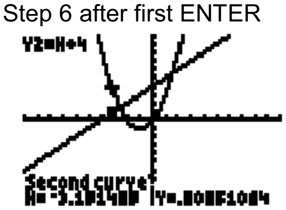
Step 5 after "5"



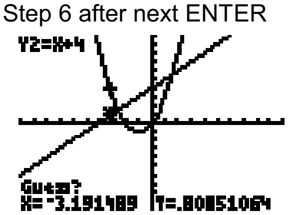
Step 6 after left arrow



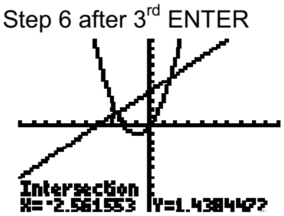
Step 6 after first ENTER



Step 6 after next ENTER



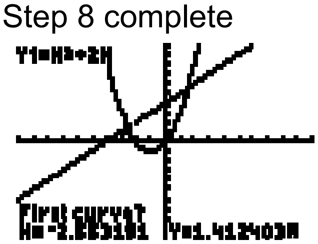
Step 6 after 3rd ENTER



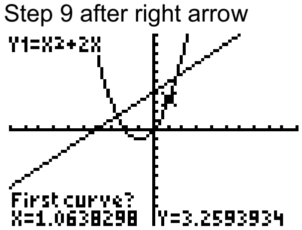
Step 7

We can read the first solution from the screen: x = -2.56 approximately

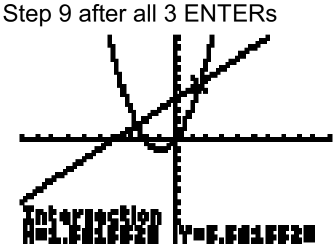
Step 8 complete



Step 9 after right arrow



Step 9 after all 3 ENTERs



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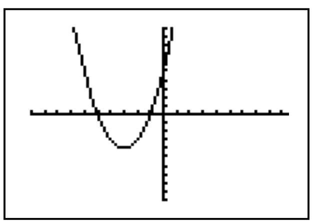
Step 10

We can read the second solution from the screen: $x = 1.56$ (approximately)

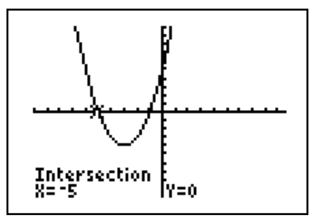
If the right-hand side of your equation is just a 0, you are looking for the points where the graph of the left side of the equation meets the x-axis. Just put in y = 0 for the second relation, and do exactly what was described above.

Example 2: Solve $x^2 + 6x + 5 = 0$

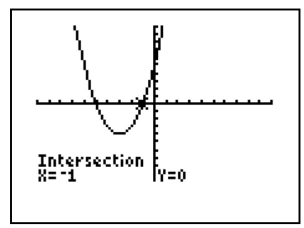
Your graph should look like this.



After step 6, you should see this.



After step 9, you should see this.



From the graphs, you can see that the solutions to the equation are x = -5 and $x = -1$.

Topic 4: Solving Quadratic Equation Word Problems

When you are given an application problem related to quadratic relations, there is one question that you must always ask:

> “Have I been given **input** values to work with or have I been given output values to work with?”

If you have been given **input** values to work with, all you have to do is substitute those values into the given quadratic function and answer the question asked.

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Example 1: Word Problem with Input values

The quadratic relation $h = 200 - 4.9t^2$ gives the height above the ground in metres for an object that is dropped from a height of 200 metres. The variable,h, is the output variable measured in metres and the variable, t, is the input variable, which measures the time in seconds after the object is dropped.

[a] How high is the object after it has been falling for 4 seconds?

We have been given a value for time, so we just substitute t = 4 into the function.

$$\begin{aligned} h &= 200 - 4.9(4)^2 \\ &= 200 - 4.9(16) \\ &= 200 - 78.4 \\ &= 121.6 \end{aligned}$$

The height of the object would be 121.6 m at a time of 4 seconds.

[b] How far does the object fall during the first 6 seconds?

Again we have been given a value for time, which is the input, so we just substitute t = 6 into the function.

$$\begin{aligned} h &= 200 - 4.9(6)^2 \\ &= 200 - 4.9(36) \\ &= 200 - 176.4 \\ &= 23.6 \text{ m} \end{aligned}$$

We have to think a bit now to answer the question asked! The function tells us where the object is, but **<u>not</u>** how far it is has fallen. We know that the object has a height of 23.6 m, which is 176.4 m lower than its starting position of 200 m.

The object falls 176.4 m during the first 6 seconds.

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Example 2: Word Problem with Output values

Remember that to solve a word problem, we should ask, “Have I been given specific input values to work with or have I been given output values to work with? ” If the answer to this question is **output**, always **replace the variable in front of the “=”** with the value that was specified in the question.

We will use the same example. Remember, our height function was $\mathbf{h = 200 - 4.9t^2}$.

[a] How long would it take the object to fall to a height of 110 m?

In this case, we have been **given** a value for height, which is **output,** and **asked** for a value for t, which is input.

We start by replacing the “h” in the function with the value given.

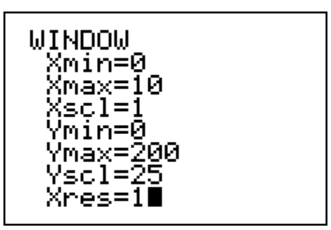
$$110 = 200 - 4.9t^2$$

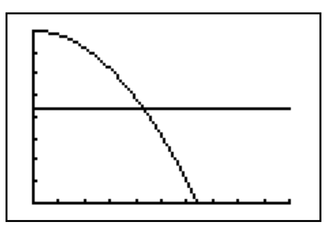
This is a quadratic equation, so we can input both sides of it into a graphing calculator to see what happens. Before we start, we should think about what times are meaningful and what heights are possible.

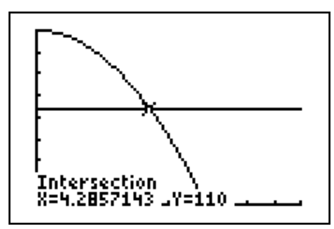
From the previous examples, we know that the object fell 176.4 metres in 6 seconds, so we know that the meaningful times to think about are from 0 seconds to, at most, 10 seconds.

We also know that the object starts at a height of 200 m and falls to the ground, which is a height of 0 m.

We use these numbers to set the window on the calculator.
Following the routine from the beginning of the lesson, here are the key screens.







The time shown on the graph is t = 4.3 seconds. To directly answer the question asked, we would say, “It would take the object about 4.3 seconds to fall to a height of 110 m.”

We obtained only one intersection point instead of two because the left-hand side of the parabola didn’t get graphed. (Even if we did graph it, we’d get a negative value for time, which we would have to reject because it would not be possible.)

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Example 2, continued

[b] About how long would it take the object to reach the ground?

We could repeat the method from part [a], replacing the 110 with a 0. Because the question asked “about how long,” this suggests that an approximate answer for time will do.

If we look at the middle screen at the bottom of the previous page, we can see that the curve that represents the height of the object reaches the bottom of the graph between 6 and 7 seconds and appears to be a bit closer to the 6 than the 7.

A quick answer would be that it will take a little bit less than 6.5 seconds for the object to reach the ground.

Assessment and Evaluation: Unit 3, Lesson 3

1. The function h = $-4.9x^2 + 40x + 1$ gives the height of a baseball after it has been hit until it lands on the ground.
The variable, h, gives the height of the ball in metres.
The variable, *x*, gives the time in seconds after the ball has been hit.

[a] Use the TABLE function of your graphing calculator to show a table of values for the height of the ball until it hits the ground.

Time in seconds	Height in metres
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

[b] What is the height of the ball at the point when it is hit? Explain.

[c] What is the height of the ball 3 seconds after it is hit?

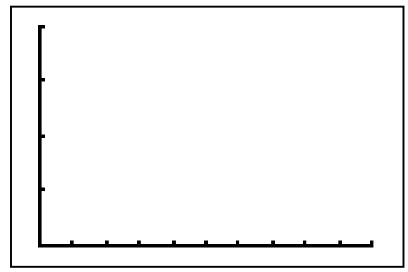
[d] Set the window of the calculator so that the graph you get will show all values for the table above. Then use your calculator to answer the following questions. Round all times to one decimal place.

i. Write the quadratic equation that can be used to find the times when the ball has a height of 40 m.

Assessment and Evaluation: Unit 3, Lesson 3

1.[d] continued

ii. Sketch the screen that shows the time at which the ball first reaches a height of 40 m. Label the scales. At what time will this happen?



iii. What is the second time at which the ball is at a height of 40 m?

iv. What is the **interval** at which the height of the ball will be greater than 40 metres?

v. Set up and solve a quadratic equation to find the time at which the ball reaches the ground. Clearly describe the steps you took to get your answer. Remember that height = 0 at ground level.

Assessment and Evaluation: Unit 3, Lesson 3

2. The relation $h = -5(t - 0.5)^2 + 11.25$ was introduced in Unit 3, Lesson 1 to describe the height of a diver above the water. The height above the water (in metres) is given by "h," while the time in seconds is given by "t."

   [a] What is the highest height that the diver will reach, and when will it happen?

   [b] According to this relation, what would the height be after 3 seconds? Explain why this value is not likely to be accurate.

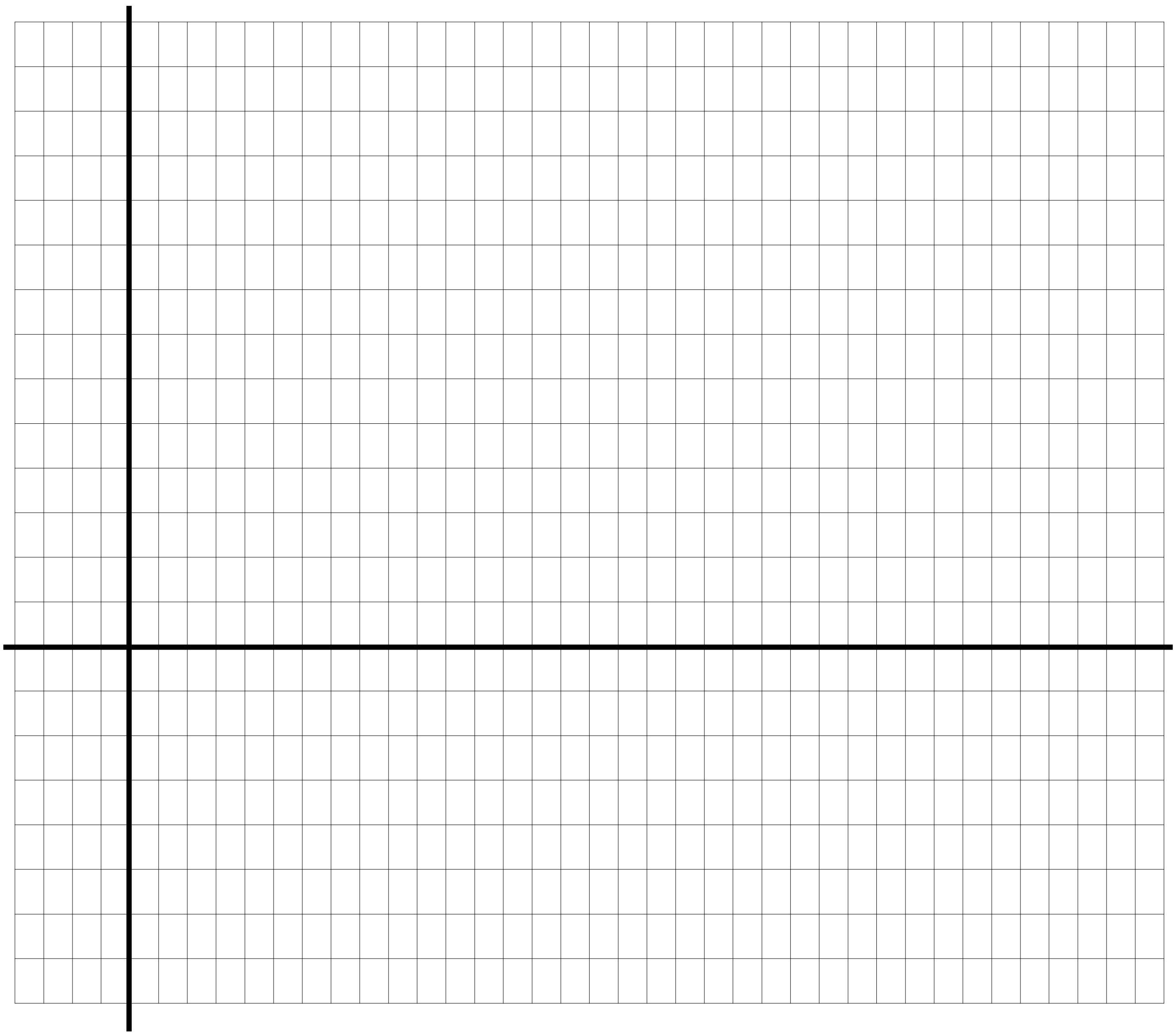
   [c] Set up and solve a quadratic equation to find the time at which the diver will hit the surface of the water. Round your answer to one decimal place.

   [d] An Olympic diving pool must be at least 5 m deep, which is probably a good idea if a person is diving from a height of 10 m and is expected to dive deep to reduce the splash. On the grid provided on the following page, **draw a neat, accurate, and fully labelled graph** showing the height of the diver, using the relation until the diver hits the water. Then use your imagination and reasoning to sketch a reasonable graph for the time underwater until the diver surfaces.

   Before you draw the graph, write a brief explanation of how you will draw the underwater portion, and why you will draw it that way.

Assessment and Evaluation: Unit 3, Lesson 3

Use a **vertical scale** of **1 square per metre** to show the height above or below the water's surface. Use a **horizontal scale** of **5 squares per second** to show the time elapsed.



Student Instruction Sheet: Unit 3, Lesson 3

Unit 3, Reflective Activity

Suggested Time: 35 minutes

What's important in this lesson:

Work carefully through the attached questions. These questions have been designed to review the topics you learned in Unit 3.

Complete the following steps:

1. Answer all questions provided.
2. If you have any questions, ask the teacher.
3. Check your answers with the Answer Key that your teacher has.

Hand in the following:

Reflective Activity

1. Draw a graph of quadratic equation that would show a maximum value of the height of a soccer ball that was kicked from the ground into the air, then fell to the ground again.

Show the height on the vertical scale.

On the horizontal scale, show the times the soccer ball is on the ground (height =0)
Hint: the ball is kicked at time=0.

2. Write an approximate quadratic equation for this graph.

Questions for the teacher: